An overview of Ampère’s Law

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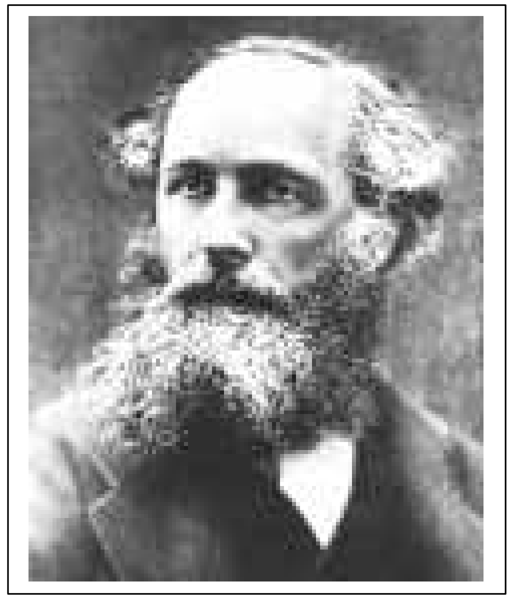
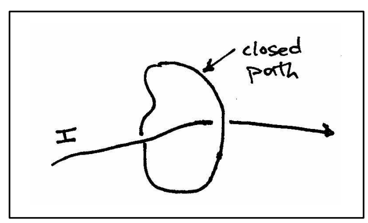


James Clerk Maxwell

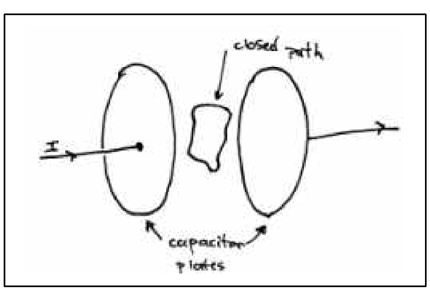
Two great guys – one great law!

$$\oint \mathbf{B} \cdot d\vec{\ell} = m_o I + m_o e_o \frac{d}{dt} f_E$$

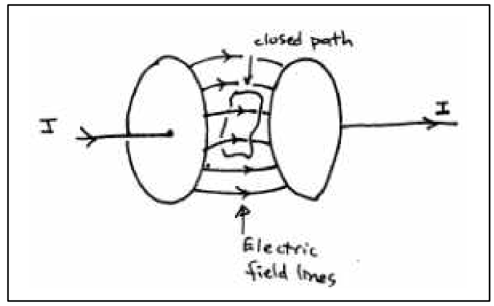
Maxwell’s four equations give the relationships between the electric field **E**, the magnetic field **B**, and charge (either stationary, denoted Q, or moving, denoted by the electric current I). In the early 1800’s, Ampère had discovered the relation $\oint \mathbf{B} \cdot d\vec{\ell} = m_o I$. The left side of this is called the “circulation of **B**”, and represents the average value of the magnetic field **B** pointing along any closed path, while on the right side, m_o (pronounced “myu-nawt”) is a fundamental constant, and the symbol I represents the electrical current which threads through the closed path, as shown to the right.



In the late 1800’s, Maxwell realized that this formulation could not be complete. He considered the example of a capacitor which is being charged up by a current I flowing onto the left plate and away from the right plate. The magnetic field **B** between the capacitor plates is non-zero, and the left side of Ampère’s law, $\oint \mathbf{B} \cdot d\vec{\ell}$, is non-zero. However, the current I clearly does not thread through the path, as shown to the left. Thus, the right side of Ampère’s law, $m_o I$ is zero and so the equation doesn’t hold. Maxwell realized that this problem could be fixed by adding a



second term to the right side, μ_o times the "displacement current" $\varepsilon_o \frac{d}{dt}\Phi_E$. Here, ε_o is another fundamental constant (pronounced "epsilon nawt"), Φ_E (pronounced "feye sub E") is the flux of the electric field **E** (essentially the number of electric field lines which thread the closed path) and $\frac{d}{dt}\Phi_E$ is the time derivitive of Φ_E, i.e. the rate at which Φ_E changes. As the capacitor charges, the electric field between the plates gets stronger, so $\frac{d}{dt}\Phi_E$ is non-zero.



This correction completed Ampère's law, the last of the four Maxwell's equations. However, it did much more than explain the magnetic field between capacitor plates! The corrected form of Ampère's law says that a changing electric field can *create* a changing magnetic field! It was already known from Faraday's law (another of Maxwell's four equations, $\oint \mathbf{E}\cdot d\vec{\ell} = -\frac{d\Phi_M}{dt}$ where Φ_M is the magnetic flux) that a changing magnetic field can create an electric field. Maxwell realized that the combination of these two laws meant that it was possible for a *self-sustaining* electromagnetic wave to propagate through a vacuum – the changing electric field creates a changing magnetic field, which creates a changing electric field, etc. However, Maxwell showed that this wave could only exist if it moved at a particular speed: $1/\sqrt{\mu_o\varepsilon_o}$. Plugging in the well-known values for the two fundamental constants μ_o and ε_o gives a speed of 186,282 miles per second – the speed of light!! Thus, Maxwell had discovered that light was actually an electromagnetic wave that could travel through vacuum.